Exercise 50

The equation of motion of a particle is $s = t^4 - 2t^3 + t^2 - t$, where s is in meters and t is in seconds.

(a) Find the velocity and acceleration as functions of t.

(b) Find the acceleration after 1 s.

(c) Graph the position, velocity, and acceleration functions on the same screen.

Solution

The velocity is the derivative of position.

$$\begin{aligned} v(t) &= \frac{ds}{dt} \\ &= \frac{d}{dt}(t^4 - 2t^3 + t^2 - t) \\ &= \frac{d}{dt}(t^4) - \frac{d}{dt}(2t^3) + \frac{d}{dt}(t^2) - \frac{d}{dt}(t) \\ &= \frac{d}{dt}(t^4) - 2\frac{d}{dt}(t^3) + \frac{d}{dt}(t^2) - \frac{d}{dt}(t) \\ &= (4t^3) - 2(3t^2) + (2t) - (1) \\ &= 4t^3 - 6t^2 + 2t - 1 \end{aligned}$$

The acceleration is the derivative of velocity.

$$\begin{aligned} a(t) &= \frac{dv}{dt} \\ &= \frac{d}{dt}(4t^3 - 6t^2 + 2t - 1) \\ &= \frac{d}{dt}(4t^3) - \frac{d}{dt}(6t^2) + \frac{d}{dt}(2t) - \frac{d}{dt}(1) \\ &= 4\frac{d}{dt}(t^3) - 6\frac{d}{dt}(t^2) + 2\frac{d}{dt}(t) - \frac{d}{dt}(1) \\ &= 4(3t^2) - 6(2t) + 2(1) - (0) \\ &= 12t^2 - 12t + 2 \end{aligned}$$

The acceleration after 1 second is

$$a(1) = 2.$$

Below is a graph of $s(t)$ versus t, $v(t)$ versus t, and $a(t)$ versus t.